# Heat Conduction in a Circle

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http://www.giacomo.lorenzoni.name/PEEI_4.0.0.1/Heat_conduction_in_a_circle/

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# Heat conduction in a circle

This text is integrating part of the homonymous link in PEEI: a computer program for the numerical solution of systems of partial differential equations.

**Coordinate system:** polar

**System of measurement:** International System of Units

**Coordinates of polar system:** $\{\rho,\alpha\}$ of which $[\rho]\equiv$[length] $[\alpha]\equiv$[plane angle] $\Re\langle\rho\rangle\equiv[0,\infty)$ $\Re\langle\alpha\rangle\equiv[0,2\cdot\pi)$

**Unknown functions:** $T(\rho,\alpha)$ of which $[T]\equiv$[temperature]

**Differential analytic model:** $\rho\cdot(\partial^2T(\rho,\alpha)/\partial\rho^2)+\partial T(\rho,\alpha)/\partial\rho+\rho\cdot Q/K=0$ of which $Q=100$ $K=20$.

**Definition set:** $\{\{\rho,\alpha\} / \rho\leq\rho_2\}$ $\rho_2=10$.

**Case 1-1:**
***Conditions:*** $T(0,\alpha)=1000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv T(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
***Related files:*** mad-A.txt, points-1.txt, PEEI-mem-1.bin, cond-1-1.txt, PEEI-sol-1-1.txt, plot-1-1.jpg

**Case 1-2:**
***Conditions:*** $T(0,\alpha)=1000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv T(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
***Related files:*** mad-A.txt, points-2.txt, PEEI-mem-2.bin, cond-1-2.txt, PEEI-sol-1-2.txt, plot-1-2.jpg

**Case 1-3:**
***Conditions:*** $T(0,\alpha)=1000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv T(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
***Related files:*** mad-A.txt, points-3.txt, PEEI-mem-3.bin, cond-1-3.txt, PEEI-sol-1-3.txt, plot-1-3.jpg

**Case 1-4:**
***Conditions:*** $T(0,\alpha)=1000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv T(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
***Related files:*** mad-A.txt, points-4.txt, PEEI-mem-4.bin, cond-1-4.txt, PEEI-sol-1-4.txt, plot-1-4.jpg

**Case 1-5:**
***Conditions:*** $T(0,\alpha)=1000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv T(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
***Related files:*** mad-A.txt, points-5.txt, PEEI-mem-5.bin, cond-1-5.txt, PEEI-sol-1-5.txt, plot-1-5.jpg

**Case 2-1:**
***Conditions:*** $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-A.txt, points-1.txt, PEEI-mem-1.bin, cond-2-1.txt, PEEI-sol-2-1.txt, plot-2-1.jpg

**Case 2-2:**
***Conditions:*** $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-A.txt, points-2.txt, PEEI-mem-2.bin, cond-2-2.txt, PEEI-sol-2-2.txt, plot-2-2.jpg

**Case 2-3:**
***Conditions:*** $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-A.txt, points-3.txt, PEEI-mem-3.bin, cond-2-3.txt, PEEI-sol-2-3.txt, plot-2-3.jpg

**Case 2-4:**
***Conditions:*** $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-A.txt, points-4.txt, PEEI-mem-4.bin, cond-2-4.txt, PEEI-sol-2-4.txt, plot-2-4.jpg

**Case 2-5:**
***Conditions:*** $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-A.txt, points-5.txt, PEEI-mem-5.bin, cond-2-5.txt, PEEI-sol-2-5.txt, plot-2-5.jpg

**Case 3-1:**
***Conditions:*** $K\cdot(\partial T(\rho_2,\alpha)/\partial\rho)+H\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $H=50$ $T_\infty=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-B.txt, points-1.txt, PEEI-mem-1.bin, cond-3-1.txt, PEEI-sol-3-1.txt, plot-3-1.jpg

**Case 3-2:**
***Conditions:*** $K\cdot(\partial T(\rho_2,\alpha)/\partial\rho)+H\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $H=50$ $T_\infty=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-B.txt, points-2.txt, PEEI-mem-2.bin, cond-3-2.txt, PEEI-sol-3-2.txt, plot-3-2.jpg

**Case 3-3:**
***Conditions:*** $K\cdot(\partial T(\rho_2,\alpha)/\partial\rho)+H\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $H=50$ $T_\infty=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-B.txt, points-3.txt, PEEI-mem-3.bin, cond-3-3.txt, PEEI-sol-3-3.txt, plot-3-3.jpg

**Case 3-4:**
***Conditions:*** $K\cdot(\partial T(\rho_2,\alpha)/\partial\rho)+H\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $H=50$ $T_\infty=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-B.txt, points-4.txt, PEEI-mem-4.bin, cond-3-4.txt, PEEI-sol-3-4.txt, plot-3-4.jpg

**Case 3-5:**
***Conditions:*** $K\cdot(\partial T(\rho_2,\alpha)/\partial\rho)+H\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $H=50$ $T_\infty=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
***Solution in [1]:*** $T(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
***Related files:*** mad-B.txt, points-5.txt, PEEI-mem-5.bin, cond-3-5.txt, PEEI-sol-3-5.txt, plot-3-5.jpg

**Bibliography:**
[1] H. S. CARSLAW, J. C. JAEGER, *Conduction of Heat in Solids*, second edition, Oxford University Press, 1986, London.