Heat Conduction in an Annulus

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http://www.giacomo.lorenzoni.name/PEEI_4.0.0.1/heat_conduction_in_an_annulus/

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Heat conduction in an annulus

This text is integrating part of the homonymous link in PEEI: a computer program for the numerical solution of systems of partial differential equations.

Coordinate system: polar

System of measurement: International System of Units

Coordinates of polar system: $\{\rho,\alpha\}$ of which $[\rho]\equiv$[length] $[\alpha]\equiv$[plane angle] $\Re\langle\rho\rangle\equiv[0,\infty)$ $\Re\langle\alpha\rangle\equiv[0,2\cdot\pi)$

Unknown functions: $\tau(\rho,\alpha)$ of which $[\tau]\equiv$[temperature]

Differential analytic model: $\rho\cdot(\partial^2\tau(\rho,\alpha)/\partial\rho^2)+\partial\tau(\rho,\alpha)/\partial\rho=0$

Definition set: $\{\{\rho,\alpha\}\ /\rho_1\leq\rho\leq\rho_2\}$ $\rho_1=1$ $\rho_2=10$

Case 1-1:
Conditions: $\tau(\rho_1,\alpha)=1000$ $\tau(\rho_2,\alpha)=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-1.txt, PEEI-mem-1.bin, cond-1-1.txt, PEEI-sol-1-1.txt, plot-1-1.jpg

Case 1-2:
Conditions: $\tau(\rho_1,\alpha)=1000$ $\tau(\rho_2,\alpha)=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-2.txt, PEEI-mem-2.bin, cond-1-2.txt, PEEI-sol-1-2.txt, plot-1-2.jpg

Case 1-3:
Conditions: $\tau(\rho_1,\alpha)=1000$ $\tau(\rho_2,\alpha)=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-3.txt, PEEI-mem-3.bin, cond-1-3.txt, PEEI-sol-1-3.txt, plot-1-3.jpg

Case 1-4:
Conditions: $\tau(\rho_1,\alpha)=1000$ $\tau(\rho_2,\alpha)=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-4.txt, PEEI-mem-4.bin, cond-1-4.txt, PEEI-sol-1-4.txt, plot-1-4.jpg

Case 1-5:
Conditions: $\tau(\rho_1,\alpha)=1000$ $\tau(\rho_2,\alpha)=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-5.txt, PEEI-mem-5.bin, cond-1-5.txt, PEEI-sol-1-5.txt, plot-1-5.jpg

Case 2-1:
Conditions: $\tau(\rho_1,\alpha)=0$ $\tau(\rho_2,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-1.txt, PEEI-mem-1.bin, cond-2-1.txt, PEEI-sol-2-1.txt, plot-2-1.jpg

Case 2-2:
Conditions: $\tau(\rho_1,\alpha)=0$ $\tau(\rho_2,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-2.txt, PEEI-mem-2.bin, cond-2-2.txt, PEEI-sol-2-2.txt, plot-2-2.jpg

Case 2-3:
Conditions: $\tau(\rho_1,\alpha)=0$ $\tau(\rho_2,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-3.txt, PEEI-mem-3.bin, cond-2-3.txt, PEEI-sol-2-3.txt, plot-2-3.jpg

Case 2-4:
Conditions: $\tau(\rho_1,\alpha)=0$ $\tau(\rho_2,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-4.txt, PEEI-mem-4.bin, cond-2-4.txt, PEEI-sol-2-4.txt, plot-2-4.jpg

Case 2-5:
Conditions: $\tau(\rho_1,\alpha)=0$ $\tau(\rho_2,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot\ln(\rho_2/\rho)+\tau(\rho_2,\alpha)\cdot\ln(\rho/\rho_1))/\ln(\rho_2/\rho_1)$
Related files: mad-A.txt, points-5.txt, PEEI-mem-5.bin, cond-2-5.txt, PEEI-sol-2-5.txt, plot-2-5.jpg

Case 3-1:
Conditions: $\tau(\rho_1,\alpha)=1000$ $\partial\tau(\rho_2,\alpha)/\partial\rho+h\cdot(\tau(\rho_2,\alpha)-\tau_\infty)=0$ $h=2.5$ $\tau_\infty=10$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho))+h\cdot\rho_2\cdot\tau_\infty\cdot\ln(\rho/\rho_1))/(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho_1))$
Related files: mad-B.txt, points-1.txt, PEEI-mem-1.bin, cond-3-1.txt, PEEI-sol-3-1.txt, plot-3-1.jpg

Case 3-2:
Conditions: $\tau(\rho_1,\alpha)=1000$ $\partial\tau(\rho_2,\alpha)/\partial\rho+h\cdot(\tau(\rho_2,\alpha)-\tau_\infty)=0$ $h=2.5$ $\tau_\infty=10$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $\tau(\rho,\alpha)\equiv(\tau(\rho_1,\alpha)\cdot(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho))+h\cdot\rho_2\cdot\tau_\infty\cdot\ln(\rho/\rho_1))/(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho_1))$
Related files: mad-B.txt, points-2.txt, PEEI-mem-2.bin, cond-3-2.txt, PEEI-sol-3-2.txt, plot-3-2.jpg

Case 3-3:
Conditions: $T(\rho_1,\alpha)=1000$ $\partial T(\rho_2,\alpha)/\partial\rho+h\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $h=2.5$ $T_\infty=10$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv(T(\rho_1,\alpha)\cdot(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho))+h\cdot\rho_2\cdot T_\infty\cdot\ln(\rho/\rho_1))/(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho_1))$
Related files: mad-B.txt, points-3.txt, PEEI-mem-3.bin, cond-3-3.txt, PEEI-sol-3-3.txt, plot-3-3.jpg

Case 3-4:
Conditions: $T(\rho_1,\alpha)=1000$ $\partial T(\rho_2,\alpha)/\partial\rho+h\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $h=2.5$ $T_\infty=10$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv(T(\rho_1,\alpha)\cdot(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho))+h\cdot\rho_2\cdot T_\infty\cdot\ln(\rho/\rho_1))/(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho_1))$
Related files: mad-B.txt, points-4.txt, PEEI-mem-4.bin, cond-3-4.txt, PEEI-sol-3-4.txt, plot-3-4.jpg

Case 3-5:
Conditions: $T(\rho_1,\alpha)=1000$ $\partial T(\rho_2,\alpha)/\partial\rho+h\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $h=2.5$ $T_\infty=10$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv(T(\rho_1,\alpha)\cdot(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho))+h\cdot\rho_2\cdot T_\infty\cdot\ln(\rho/\rho_1))/(1+h\cdot\rho_2\cdot\ln(\rho_2/\rho_1))$
Related files: mad-B.txt, points-5.txt, PEEI-mem-5.bin, cond-3-5.txt, PEEI-sol-3-5.txt, plot-3-5.jpg

Case 4-1:
Conditions: $T(\rho_1,\alpha)=1000$ $\{Q=-2\cdot\pi\cdot\rho\cdot K\cdot(\partial T(\rho,\alpha)/\partial\rho);\forall\rho_1<\rho<\rho_2\}$ $K=20$ $Q=10000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv T(\rho_1,\alpha)-Q\cdot\ln(\rho/\rho_1)/(2\cdot\pi\cdot K)$
Related files: mad-A.txt, points-1.txt, PEEI-mem-1.bin, cond-4-1.txt, PEEI-sol-4-1.txt, plot-4-1.jpg

Case 4-2:
Conditions: $T(\rho_1,\alpha)=1000$ $\{Q=-2\cdot\pi\cdot\rho\cdot K\cdot(\partial T(\rho,\alpha)/\partial\rho);\forall\rho_1<\rho<\rho_2\}$ $K=20$ $Q=10000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv T(\rho_1,\alpha)-Q\cdot\ln(\rho/\rho_1)/(2\cdot\pi\cdot K)$
Related files: mad-A.txt, points-2.txt, PEEI-mem-2.bin, cond-4-2.txt, PEEI-sol-4-2.txt, plot-4-2.jpg

Case 4-3:
Conditions: $T(\rho_1,\alpha)=1000$ $\{Q=-2\cdot\pi\cdot\rho\cdot K\cdot(\partial T(\rho,\alpha)/\partial\rho);\forall\rho_1<\rho<\rho_2\}$ $K=20$ $Q=10000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv T(\rho_1,\alpha)-Q\cdot\ln(\rho/\rho_1)/(2\cdot\pi\cdot K)$
Related files: mad-A.txt, points-3.txt, PEEI-mem-3.bin, cond-4-3.txt, PEEI-sol-4-3.txt, plot-4-3.jpg

Case 4-4:
Conditions: $T(\rho_1,\alpha)=1000$ $\{Q=-2\cdot\pi\cdot\rho\cdot K\cdot(\partial T(\rho,\alpha)/\partial\rho);\forall\rho_1<\rho<\rho_2\}$ $K=20$ $Q=10000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv T(\rho_1,\alpha)-Q\cdot\ln(\rho/\rho_1)/(2\cdot\pi\cdot K)$
Related files: mad-A.txt, points-4.txt, PEEI-mem-4.bin, cond-4-4.txt, PEEI-sol-4-4.txt, plot-4-4.jpg

Case 4-5:
Conditions: $\mathrm{T}(\rho_1,\alpha)=1000$ $\{Q=-2\cdot\pi\cdot\rho\cdot K\cdot(\partial T(\rho,\alpha)/\partial\rho);\forall\rho_1<\rho<\rho_2\}$ $K=20$ $Q=10000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution by [1]: $T(\rho,\alpha)\equiv T(\rho_1,\alpha)-Q\cdot\ln(\rho/\rho_1)/(2\cdot\pi\cdot K)$
Related files: mad-A.txt, points-5.txt, PEEI-mem-5.bin, cond-4-5.txt, PEEI-sol-4-5.txt, plot-4-5.jpg

Bibliography:
[1] H. S. CARSLAW, J. C. JAEGER, *Conduction of Heat in Solids*, second edition, Oxford University Press, 1986, London.