The solution of a combinatorial problem.

GIACOMO LORENZONI

Abstract
A procedure is exposed in order to determine the solution of a combinatorial problem. The data of the problem are a natural $\boldsymbol{N}$ *and a positive real* $\boldsymbol{\Delta}$*. The solution of the problem are* $\boldsymbol{N}$ *positive real such that the succession of the* $2^{\boldsymbol{N}}-1$ *sums, each defined by possessing like addends the elements of one of the as many combinations of the* $\boldsymbol{N}$ *numbers, it can increasingly be ordered turning out that the* $2^{\boldsymbol{N}}-2$ *successive increments are all equal to* $\boldsymbol{\Delta}$*.*

Keywords: *combinatorics; combinatorial analysis; necessity and sufficiency; deducibility; problem resolution.*

1. PRELIMINARY POSITIONS.

The Author released the italian version of this script, officially and in its present definitive form, in date 18/09/2003.

A name of a object is a symbol that refers it and marks it, in how much it attributes to it some properties whose totality is only possessed from it. It is used the symbol ≡ in the sense that a writing A≡B means that A and B are two different names of a same object. A A≡B is used also to call a object whose name is A and of whom is valid the A≡B.

By intending that $\mathcal{P}_A$ $\mathcal{P}_B$ and $\mathcal{P}$ are three propositions (and that a proposition can be constituted from a set of propositions); are used: the $\{\mathcal{P}_A \,|\, \mathcal{P}_B\}$ for to refer the proposition who is obtained by subjecting $\mathcal{P}_A$ to all the conditions induced from $\mathcal{P}_B$, and the $\mathcal{P}_A \Rightarrow \mathcal{P}_B$ for to indicate that from $\mathcal{P}_A$ is deducible $\mathcal{P}_B$; and are intended the $\{\mathcal{P}\} \equiv \{\mathcal{P}$ is true$\}$ $\neg\{\mathcal{P}\} \equiv \{\mathcal{P}$ is false$\}$.

It is indicated $\underline{\boldsymbol{\mathcal{P}}}\langle\mathcal{P}_A,\mathcal{P}_B\rangle$ a set of propositions from which is deducible the proposition $\mathcal{P}_A$ and such that they are all true except the $\mathcal{P}_B$ that can be true or false.

Are admitted the

$$\{\mathcal{P}_A\} \Rightarrow \{\mathcal{P}_B\} \quad \neg\{\mathcal{P}_B\} \Rightarrow \neg\{\mathcal{P}_A\} \tag{1.1}$$

valid if is known the existence of a $\underline{\boldsymbol{\mathcal{P}}}\langle\mathcal{P}_B,\mathcal{P}_A\rangle$. This existence implies: the first of the (1.1), because if $\{\mathcal{P}_A\}$ then from $\underline{\boldsymbol{\mathcal{P}}}\langle\mathcal{P}_B,\mathcal{P}_A\rangle$ is deduced $\{\mathcal{P}_B\}$; the second of the (1.1), because if $\neg\{\mathcal{P}_B\}$ then $\{\mathcal{P}_B\}$ is not deducible from $\underline{\boldsymbol{\mathcal{P}}}\langle\mathcal{P}_B,\mathcal{P}_A\rangle$ and this impossibility can be had to alone to $\neg\{\mathcal{P}_A\}$.

Are admitted the

$$\{\{\mathcal{P}_A\} \Rightarrow \{\mathcal{P}_B\}\} \equiv \{\{\mathcal{P}_A\} \text{ is sufficient for } \{\mathcal{P}_B\}\} \tag{1.2}$$

$$\{\neg\{\mathcal{P}_B\} \Rightarrow \neg\{\mathcal{P}_A\}\} \equiv \{\{\mathcal{P}_B\} \text{ is necessary for } \{\mathcal{P}_A\}\} \tag{1.3}$$

The (1.2) is admitted because: the deducibility of $\{\mathcal{P}_B\}$ from $\{\mathcal{P}_A\}$ implies that this is sufficient for that one; the sufficiency of $\{\mathcal{P}_A\}$ for $\{\mathcal{P}_B\}$ implies the existence of a $\underline{\boldsymbol{\mathcal{P}}}\langle\mathcal{P}_B,\mathcal{P}_A\rangle$ and hence of the $\{\mathcal{P}_A\} \Rightarrow \{\mathcal{P}_B\}$.

The (1.3) is admitted because: the deducibility of $\neg\{\mathcal{P}_A\}$ from $\neg\{\mathcal{P}_B\}$ implies that $\{\mathcal{P}_A\}$

cannot exist when $\neg\{\mathcal{P}_B\}$ exists and hence that $\{\mathcal{P}_B\}$ is necessary for $\{\mathcal{P}_A\}$; the being $\{\mathcal{P}_B\}$ necessary for $\{\mathcal{P}_A\}$ implies that its absence, i.e. $\neg\{\mathcal{P}_B\}$, imposes $\neg\{\mathcal{P}_A\}$ by following from this the $\neg\{\mathcal{P}_B\}\Rightarrow\neg\{\mathcal{P}_A\}$.

The (1.1) and the said condition which makes them valid, and the {(1.2),(1.3)}; give place to the

$$\{\{\{\mathcal{P}_A\}\text{ is sufficient for }\{\mathcal{P}_B\}\}\ \text{.and.}\ \{\{\mathcal{P}_B\}\text{ is necessary for }\{\mathcal{P}_A\}\};\forall\underline{\mathcal{P}}\langle\mathcal{P}_B,\mathcal{P}_A\rangle\} \quad (1.4)$$

It is indicated $\{c=A,B\}$ with A and B two integers that verify the $A\leq B$, the succession of the $B-A+1$ integers that increase orderly from A to B, i.e. it is intended the $\{c=A,B\}\equiv\{A,A+1,\ldots,B\}$. In accordance with this, are intended the

$\{D_c;c=A,B\}\equiv\{D_A,D_{A+1},\ldots,D_B\}$ $\Sigma_{c=A,B}(D_c)\equiv\{D_A+D_{A+1}+\ldots+D_B\}$ $\{\Sigma_{c=A,B}(D_c)\equiv0;\forall\{B<A\}\}$ $A\text{-}_{ab\ldots c}\equiv A_{a,b,\ldots,c}\equiv A\langle a,b,\ldots,c\rangle$
$\{\S_{\underline{i}};i_1=A_1,B_1;i_2=A_2,B_2;\ldots;i_{ɉ}=A_{ɉ},B_{ɉ}\}\equiv\{\{\ldots\{\{\S_{\underline{i}};i_1=A_1,B_1\};i_2=A_2,B_2\};\ldots\};i_{ɉ}=A_{ɉ},B_{ɉ}\}$

where $\underline{i}\equiv\{i_j;j=1,ɉ\}$.

It is indicated $Б\langle N,K\rangle\equiv N!/((N-K)!\cdot K!)$ the number of combinations of class K of N objects (as in the following text, it is intended only combinations without repetition), and it is put the $\mathbf{Б}\langle N\rangle\equiv\Sigma_{n=1,N}(Б_{Nn})$ whose is valid the known

$$\mathbf{Б}_N=2^N-1=\Sigma_{n=1,N}(2^{n-1}) \quad (1.5)$$

It is called $\underline{n}\langle c,b,a\rangle$ the a-th element of the b-th combination of class c of the $\{n=1,N\}$, by following from this both the $c\in\{n=1,N\}$ $b\in\{b=1,Б_{Nc}\}$ $a\in\{a=1,c\}$ and that $\{\underline{n}_{cba};a=1,c\}$ is the b-th combination of class c of such $\{n=1,N\}$.

2. The constitution of the problem.

The problem is constituted from: the data, i.e. the priority conditions known and not modifiable in the context of an its particular treatment; the solution, i.e. how much it demands to determine; the enunciate, i.e. the formulation of the properties that define the solution; the resolution, i.e. the determination of the solution; the resolutive procedure, i.e. the activity that has as aim the resolution.

3. The data.

The data of the problem are one couple $\{\boldsymbol{N},\boldsymbol{\Delta}\}$, whose $\boldsymbol{N}$ is a natural who verify the $\boldsymbol{N}>1$, and whose $\boldsymbol{\Delta}$ is a real who verify the $\boldsymbol{\Delta}>0$.

4 The enunciate.

It is called $\underline{\mathcal{N}}\equiv\{\mathcal{N}_n;n=1,\boldsymbol{N}\}$ a succession of $\boldsymbol{N}$ positive real, who verify the

$$\{\mathcal{N}_{n-1}\leq\mathcal{N}_n;n=2,\boldsymbol{N}\} \quad (4.1)$$

and whose are put the

$$\underline{\mathbb{S}}_{\mathcal{N}}\equiv\{\mathbb{S}_{\mathcal{N}\mathrm{m}};\mathrm{m}=1,\mathbf{Б}_{\boldsymbol{N}}\}\equiv\{\{\mathbb{S}_{\mathcal{N}m};m=1,\mathbf{Б}_{\boldsymbol{N}}\}\mid\{\mathbb{S}_{\mathcal{N},m-1}\leq\mathbb{S}_{\mathcal{N}m};m=2,\mathbf{Б}_{\boldsymbol{N}}\}\}\tag{4.2}$$

$$\{\Delta_{\mathbb{S}\langle\mathcal{N}\rangle\mathrm{m}}\equiv(\mathbb{S}_{\mathcal{N},\mathrm{m}+1}-\mathbb{S}_{\mathcal{N}\mathrm{m}});\mathrm{m}=1,\mathbf{Б}_{\boldsymbol{N}}-1\}\tag{4.3}$$

defined from the

$$\{\mathbb{S}_{\mathcal{N}m};m=1,\mathbf{Б}_{\boldsymbol{N}}\}\equiv\{\Sigma_{\alpha=1,\mathrm{c}}(\mathcal{N}_{\mathfrak{n}\langle\mathrm{c},\mathfrak{b},\alpha\rangle});\mathfrak{b}=1,\mathrm{Б}_{\boldsymbol{N}\mathrm{c}};\mathrm{c}=1,\boldsymbol{N}\}$$

for which are valid the

$$\mathbb{S}_{\mathcal{N}\mathbf{Б}\langle\boldsymbol{N}\rangle}\equiv\Sigma_{\mathrm{n}=1,\boldsymbol{N}}(\mathcal{N}_{\mathrm{n}})\ \{\Delta_{\mathbb{S}\langle\mathcal{N}\rangle\mathrm{m}}\geq0;\mathrm{m}=1,\mathbf{Б}_{\boldsymbol{N}}-1\}\tag{4.4}$$

The solution of the problem is a succession $\underline{\mathcal{N}}$ whose $\underline{\mathbb{S}}_{\mathcal{N}}$ verify the

$$\{\Delta_{\mathbb{S}\langle\mathcal{N}\rangle\mathrm{m}}=\boldsymbol{\Delta};\mathrm{m}=1,\mathbf{Б}_{\boldsymbol{N}}-1\}\tag{4.5}$$

who is cited by intending the (4.3).

5 The resolutive procedure.

5.1. *The first argumentation.*

From the (4.2) it follows the $\Sigma_{\alpha=1,\mathrm{c}}(\mathcal{N}_{\mathfrak{n}\langle\mathrm{c},\mathfrak{b},\alpha\rangle})\in\underline{\mathbb{S}}_{\mathcal{N}}$ for which the $\mathbb{S}_{\mathcal{N}\mathrm{m}}\equiv\Sigma_{\alpha=1,\mathrm{c}}(\mathcal{N}_{\mathfrak{n}\langle\mathrm{c},\mathfrak{b},\alpha\rangle})$ is used by intending it inherent an any $\mathbb{S}_{\mathcal{N}\mathrm{m}}\in\underline{\mathbb{S}}_{\mathcal{N}}$ between the $\mathbf{Б}_{\boldsymbol{N}}$ possible. From the $\Sigma_{\alpha=1,\mathrm{c}}(\mathcal{N}_{\mathfrak{n}\langle\mathrm{c},\mathfrak{b},\alpha\rangle})\in\underline{\mathbb{S}}_{\mathcal{N}}$ follows the $\{\mathcal{N}_{\mathrm{n}}\in\underline{\mathbb{S}}_{\mathcal{N}};\mathrm{n}=1,\boldsymbol{N}\}$.

The (4.1), the $\{\mathcal{N}_{\mathrm{n}}\in\underline{\mathbb{S}}_{\mathcal{N}};\mathrm{n}=1,\boldsymbol{N}\}$, and the (4.5); imply the

$$\{\mathcal{N}_{\mathrm{n}-1}<\mathcal{N}_{\mathrm{n}};\mathrm{n}=2,\boldsymbol{N}\}\tag{5.1.1}$$

The second of the (4.4), the $\Sigma_{\alpha=1,\mathrm{c}}(\mathcal{N}_{\mathfrak{n}\langle\mathrm{c},\mathfrak{b},\alpha\rangle})\in\underline{\mathbb{S}}_{\mathcal{N}}$, and the (5.1.1); imply the

$$\mathbb{S}_{\mathcal{N}1}\equiv\mathcal{N}_1\ \mathbb{S}_{\mathcal{N},\mathbf{Б}\langle\boldsymbol{N}\rangle-1}\equiv\Sigma_{\mathrm{n}=2,\boldsymbol{N}}(\mathcal{N}_{\mathrm{n}})\tag{5.1.2}$$

The first of the (4.4), and the second of the (5.1.2); imply the $\mathbb{S}_{\mathcal{N}\mathbf{Б}\langle\boldsymbol{N}\rangle}-\mathbb{S}_{\mathcal{N},\mathbf{Б}\langle\boldsymbol{N}\rangle-1}=\mathcal{N}_1$. This, and the $\mathbb{S}_{\mathcal{N}\mathbf{Б}\langle\boldsymbol{N}\rangle}-\mathbb{S}_{\mathcal{N},\mathbf{Б}\langle\boldsymbol{N}\rangle-1}=\boldsymbol{\Delta}$ who follows from the (4.5); imply the $\mathcal{N}_1=\boldsymbol{\Delta}$.

It is called $\mathbb{S}_{\mathcal{N}K}$ the unique specification of the generic $\mathbb{S}_{\mathcal{N}\mathrm{m}}\equiv\Sigma_{\alpha=1,\mathrm{c}}(\mathcal{N}_{\mathfrak{n}\langle\mathrm{c},\mathfrak{b},\alpha\rangle})$ that it is possible in correspondence of a $\mathbb{S}_{\mathcal{N},K-1}\equiv\Sigma_{\mathrm{n}=1,\mathrm{k}-1}(\mathcal{N}_{\mathrm{n}})$. The (4.5), and the (5.1.1); imply that a such sum $\mathbb{S}_{\mathcal{N}K}$ have at least one of the $\{\mathcal{N}_{\mathrm{n}};\mathrm{n}=\mathrm{k},\boldsymbol{N}\}$ between the own addends and it is the minor between every element of $\underline{\mathbb{S}}_{\mathcal{N}}$ that has this property. These two requirements of $\mathbb{S}_{\mathcal{N}K}$, and the (5.1.1); determine it like the $\mathbb{S}_{\mathcal{N}K}\equiv\mathcal{N}_{\mathrm{k}}$.

These $\{\mathbb{S}_{\mathcal{N},K-1}\equiv\Sigma_{\mathrm{n}=1,\mathrm{k}-1}(\mathcal{N}_{\mathrm{n}}),\mathbb{S}_{\mathcal{N}K}\equiv\mathcal{N}_{\mathrm{k}}\}$ who are defined for $\mathrm{k}\in\{\mathrm{k}=2,\boldsymbol{N}\}$, and the $\mathbb{S}_{\mathcal{N}K}-\mathbb{S}_{\mathcal{N},K-1}=\boldsymbol{\Delta}$ who follows from the (4.5); imply the further $\boldsymbol{N}-1$ between the

$$\{\mathcal{N}_{\mathrm{k}}-\Sigma_{\mathrm{n}=1,\mathrm{k}-1}(\mathcal{N}_{\mathrm{n}})=\boldsymbol{\Delta};\mathrm{k}=1,\boldsymbol{N}\}\tag{5.1.3}$$

whose $\mathcal{N}_1-\Sigma_{\mathrm{n}=1,0}(\mathcal{N}_{\mathrm{n}})=\boldsymbol{\Delta}$ is the said $\mathcal{N}_1=\boldsymbol{\Delta}$.

The (5.1.3) are a linear system of $\boldsymbol{N}$ equations in the as much unknowns $\underline{\mathcal{N}}$, which admits as unique solution the

$$\underline{\mathcal{N}}\equiv\{2^{n-1}\cdot\boldsymbol{\Delta};n=1,\boldsymbol{N}\} \qquad (5.1.4)$$

5.2. *The second argumentation.*

By introducing in the $\mathbb{S}_{\underline{\mathcal{N}}m}\equiv\Sigma_{\alpha=1,c}(\mathcal{N}_{n\langle c,b,\alpha\rangle})$ the values of the $\{\mathcal{N}_{n\langle c,b,\alpha\rangle};\alpha=1,c\}$ asserted from the (5.1.4), they follow the first two members of the

$$\mathbb{S}_{\underline{\mathcal{N}}m}\equiv\Sigma_{\alpha=1,c}(2^{n\langle c,b,\alpha\rangle-1})\cdot\boldsymbol{\Delta}=\{1\,\mathbf{or}\,(2\cdot\nu_{\underline{\mathcal{N}}m})\,\mathbf{or}\,(2\cdot\nu_{\underline{\mathcal{N}}m}+1)\}\cdot\boldsymbol{\Delta} \qquad (5.2.1)$$

where $\nu_{\underline{\mathcal{N}}m}$ is a natural that verify the $1\leq\nu_{\underline{\mathcal{N}}m}\leq(2^{\boldsymbol{N}-1}-1)$.

The second of the (4.4), and the (5.2.1); allow to deduct both the $\mathbb{S}_{\underline{\mathcal{N}}1}=\boldsymbol{\Delta}$ and that $\boldsymbol{\Delta}$ is the minimum of the $\{\Delta_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m};m=1,\mathbf{Б}_{\boldsymbol{N}}-1\}$ whose (4.3) gives place to the second member of the

$$\Sigma_{m=1,\mathbf{Б}\langle\boldsymbol{N}\rangle-1}(\Delta_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m})=\Sigma_{m=1,\mathbf{Б}\langle\boldsymbol{N}\rangle-1}(\mathbb{S}_{\underline{\mathcal{N}},m+1}-\mathbb{S}_{\underline{\mathcal{N}}m})=\mathbb{S}_{\underline{\mathcal{N}}\mathbf{Б}\langle\boldsymbol{N}\rangle}-\mathbb{S}_{\underline{\mathcal{N}}1} \qquad (5.2.2)$$

The first of the (4.4), and the (5.1.4); imply the $\mathbb{S}_{\underline{\mathcal{N}}\mathbf{Б}\langle\boldsymbol{N}\rangle}=\boldsymbol{\Delta}\cdot\Sigma_{n=1,\boldsymbol{N}}(2^{n-1})$. This and the $\mathbb{S}_{\underline{\mathcal{N}}1}=\boldsymbol{\Delta}$, and the (1.5); give respectively place to the further members of the

$$\mathbb{S}_{\underline{\mathcal{N}}\mathbf{Б}\langle\boldsymbol{N}\rangle}-\mathbb{S}_{\underline{\mathcal{N}}1}=\boldsymbol{\Delta}\cdot(\Sigma_{n=1,\boldsymbol{N}}(2^{n-1})-1)=\boldsymbol{\Delta}\cdot(\mathbf{Б}_{\boldsymbol{N}}-1) \qquad (5.2.3)$$

The being $\boldsymbol{\Delta}$ the minimum of the $\{\Delta_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m};m=1,\mathbf{Б}_{\boldsymbol{N}}-1\}$ of which is valid the second of the (4.4), allow to put also the second of the

$$\{\Delta_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m}=\boldsymbol{\Delta}+\xi_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m};m=1,\mathbf{Б}_{\boldsymbol{N}}-1\}\ \{\xi_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m}\geq0;m=1,\mathbf{Б}_{\boldsymbol{N}}-1\} \qquad (5.2.4)$$

whose first gives place to the

$$\Sigma_{m=1,\mathbf{Б}\langle\boldsymbol{N}\rangle-1}(\Delta_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m})=\boldsymbol{\Delta}\cdot(\mathbf{Б}_{\boldsymbol{N}}-1)+\Sigma_{m=1,\mathbf{Б}\langle\boldsymbol{N}\rangle-1}(\xi_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m}) \qquad (5.2.5)$$

By introducing the $\{(5.2.5),(5.2.3)\}$ in the (5.2.2), it is obtained the $\Sigma_{m=1,\mathbf{Б}\langle\boldsymbol{N}\rangle-1}(\xi_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m})=0$ that, for the second of the (5.2.4), implies the $\{\xi_{\mathbb{S}\langle\underline{\mathcal{N}}\rangle m}=0;m=1,\mathbf{Б}_{\boldsymbol{N}}-1\}$. This, and the first of the (5.2.4); imply the (4.5).

5.3. *The results of the two previous argumentations.*

The (5.1.4) was deduced in the section 5.1 from a $\underline{\boldsymbol{\mathcal{P}}}_{\mathrm{I}}\equiv\underline{\mathcal{P}}_{\mathrm{I}}\,\mathbf{and}\,(4.5)$ whose $\underline{\mathcal{P}}_{\mathrm{I}}$ is a set of propositions all introduced as true (while to the (4.5) it has not been possible to attribute neither such character nor the opposite, because it was introduced as a property of the solution requested from the problem who, as such, not only it was unknown but it was also alone determinable hypothetically i.e. until contrary test).

Therefore this $\underline{\boldsymbol{\mathcal{P}}}_{\mathrm{I}}$ implies that the $\{(4.5),(5.1.4)\}$ are respective specifications of the $\{\mathcal{P}_{\mathrm{A}},\mathcal{P}_{\mathrm{B}}\}$ of which are valid the (1.1), and hence that in compliance with the (1.4) they have valid the

$$\{\{(4.5)\}\text{ is sufficient for }\{(5.1.4)\}\}\ \{\{(5.1.4)\}\text{ is necessary for }\{(4.5)\}\} \qquad (5.3.1)$$

The (4.5) was deduced in the section 5.2 from a $\underline{\mathcal{P}}_{II} \equiv \underline{\mathcal{P}}_{II}$ and (5.1.4) whose $\underline{\mathcal{P}}_{II}$ is a set of propositions all introduced like true.

Therefore this $\underline{\mathcal{P}}_{II}$ implies that the {(5.1.4),(4.5)} are respective specifications of the $\{\mathcal{P}_A, \mathcal{P}_B\}$ of which are valid the (1.1), and hence that in compliance with the (1.4) they have valid the

$$\{\{(5.1.4)\} \text{ is sufficient for } \{(4.5)\}\}\ \{\{(4.5)\} \text{ is necessary for } \{(5.1.4)\}\} \tag{5.3.2}$$

The respective expressions of the {(5.1.3),(5.1.4)} in matrix form and the known rules of such form, imply that are possible the transformation of the (5.1.3) in the (5.1.4) and the inverse transformation. Consequently the {(5.3.1),(5.3.2)} remain valid if in them is replaced the (5.1.4) with the (5.1.3), by following from this in particular valid the

$$\{\{(5.1.3)\} \text{ is necessary for } \{(4.5)\}\} \tag{5.3.3}$$

5.4. *The resolution.*

5.4.1. *The determination of a solution.*

The first of the (5.3.2) show that the (5.1.4) expresses a solution of the problem because is sufficient to ensure the property (4.5).

5.4.2. *The uniqueness of the solution.*

The system (5.1.3) is, as every system of equations, a constraint imposed on its unknowns $\underline{\mathcal{N}}$.

Such constraint consists in the being, those that are expressed from the (5.1.4), the unique possible values for the unknowns $\underline{\mathcal{N}}$. Hence the existence of a solution different from the (5.1.4) implies the violation of the constraint (5.1.3) i.e. its inefficacy.

The (5.3.3) affirms the efficacy of the constraint (5.1.3) as necessary for the subsistence of the (4.5).

Therefore the solution expressed from the (5.1.4) is unique because the existence of a solution different from it would imply the inefficacy of the constraint (5.1.3) when rather the efficacy of this is necessary for the subsistence of the (4.5).

Giacomo Lorenzoni
e-mail: info@giacomo.lorenzoni.name, info@giacomolorenzoni.com
personal site: www.giacomo.lorenzoni.name, www.giacomolorenzoni.com