Heat Conduction in a Circle

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Giacomo Lorenzoni

http://www.giacomo.lorenzoni.name/PEEI_4.0.0.1/Heat_conduction_in_a_circle/

http://www.giacomo.lorenzoni.name/PEEI_4.0.0.1/PEEIapplDown.aspx?var=2

Heat conduction in a circle

This text is integrating part of the homonymous link in PEEI: a computer program for the numerical solution of systems of partial differential equations.

Coordinate system: polar

System of measurement: International System of Units

Coordinates of polar system: $\{\rho,\alpha\}$ of which $[\rho]\equiv$[length] $[\alpha]\equiv$[plane angle] $\Re\langle\rho\rangle\equiv[0,\infty)$ $\Re\langle\alpha\rangle\equiv[0,2\cdot\pi)$

Unknown functions: $\tau(\rho,\alpha)$ of which $[\tau]\equiv$[temperature]

Differential analytic model: $\rho\cdot(\partial^2\tau(\rho,\alpha)/\partial\rho^2)+\partial\tau(\rho,\alpha)/\partial\rho+\rho\cdot Q/K=0$ of which $Q=100$ $K=20$.

Definition set: $\{\{\rho,\alpha\} / \rho\leq\rho_2\}$ $\rho_2=10$.

Case 1-1:
Conditions: $\tau(0,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $\tau(\rho,\alpha)\equiv\tau(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
Related files: mad-A.txt, points-1.txt, PEEI-mem-1.bin, cond-1-1.txt, PEEI-sol-1-1.txt, plot-1-1.jpg

Case 1-2:
Conditions: $\tau(0,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $\tau(\rho,\alpha)\equiv\tau(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
Related files: mad-A.txt, points-2.txt, PEEI-mem-2.bin, cond-1-2.txt, PEEI-sol-1-2.txt, plot-1-2.jpg

Case 1-3:
Conditions: $\tau(0,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $\tau(\rho,\alpha)\equiv\tau(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
Related files: mad-A.txt, points-3.txt, PEEI-mem-3.bin, cond-1-3.txt, PEEI-sol-1-3.txt, plot-1-3.jpg

Case 1-4:
Conditions: $\tau(0,\alpha)=1000$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $\tau(\rho,\alpha)\equiv\tau(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
Related files: mad-A.txt, points-4.txt, PEEI-mem-4.bin, cond-1-4.txt, PEEI-sol-1-4.txt, plot-1-4.jpg

Case 1-5:
Conditions: $T(0,\alpha)=1000$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv T(0,\alpha)-0.25\cdot Q\cdot\rho^2/K$
Related files: mad-A.txt, points-5.txt, PEEI-mem-5.bin, cond-1-5.txt, PEEI-sol-1-5.txt, plot-1-5.jpg

Case 2-1:
Conditions: $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
Related files: mad-A.txt, points-1.txt, PEEI-mem-1.bin, cond-2-1.txt, PEEI-sol-2-1.txt, plot-2-1.jpg

Case 2-2:
Conditions: $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
Related files: mad-A.txt, points-2.txt, PEEI-mem-2.bin, cond-2-2.txt, PEEI-sol-2-2.txt, plot-2-2.jpg

Case 2-3:
Conditions: $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
Related files: mad-A.txt, points-3.txt, PEEI-mem-3.bin, cond-2-3.txt, PEEI-sol-2-3.txt, plot-2-3.jpg

Case 2-4:
Conditions: $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
Related files: mad-A.txt, points-4.txt, PEEI-mem-4.bin, cond-2-4.txt, PEEI-sol-2-4.txt, plot-2-4.jpg

Case 2-5:
Conditions: $T(\rho_2,\alpha)=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv 0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
Related files: mad-A.txt, points-5.txt, PEEI-mem-5.bin, cond-2-5.txt, PEEI-sol-2-5.txt, plot-2-5.jpg

Case 3-1:
Conditions: $K\cdot(\partial T(\rho_2,\alpha)/\partial\rho)+H\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $H=50$ $T_\infty=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
Related files: mad-B.txt, points-1.txt, PEEI-mem-1.bin, cond-3-1.txt, PEEI-sol-3-1.txt, plot-3-1.jpg

Case 3-2:
Conditions: $K\cdot(\partial T(\rho_2,\alpha)/\partial\rho)+H\cdot(T(\rho_2,\alpha)-T_\infty)=0$ $H=50$ $T_\infty=0$ $\partial T(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $T(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/K$
Related files: mad-B.txt, points-2.txt, PEEI-mem-2.bin, cond-3-2.txt, PEEI-sol-3-2.txt, plot-3-2.jpg

Case 3-3:
Conditions: $\kappa\cdot(\partial\tau(\rho_2,\alpha)/\partial\rho)+H\cdot(\tau(\rho_2,\alpha)-\tau_\infty)=0$ $H=50$ $\tau_\infty=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $\tau(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/\kappa$
Related files: mad-B.txt, points-3.txt, PEEI-mem-3.bin, cond-3-3.txt, PEEI-sol-3-3.txt, plot-3-3.jpg

Case 3-4:
Conditions: $\kappa\cdot(\partial\tau(\rho_2,\alpha)/\partial\rho)+H\cdot(\tau(\rho_2,\alpha)-\tau_\infty)=0$ $H=50$ $\tau_\infty=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $\tau(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/\kappa$
Related files: mad-B.txt, points-4.txt, PEEI-mem-4.bin, cond-3-4.txt, PEEI-sol-3-4.txt, plot-3-4.jpg

Case 3-5:
Conditions: $\kappa\cdot(\partial\tau(\rho_2,\alpha)/\partial\rho)+H\cdot(\tau(\rho_2,\alpha)-\tau_\infty)=0$ $H=50$ $\tau_\infty=0$ $\partial\tau(\rho,\alpha)/\partial\alpha=0$
Solution in [1]: $\tau(\rho,\alpha)\equiv 0.5\cdot\rho_2\cdot Q/H+0.25\cdot Q\cdot(\rho_2^2-\rho^2)/\kappa$
Related files: mad-B.txt, points-5.txt, PEEI-mem-5.bin, cond-3-5.txt, PEEI-sol-3-5.txt, plot-3-5.jpg

Bibliography:
[1] H. S. CARSLAW, J. C. JAEGER, *Conduction of Heat in Solids*, second edition, Oxford University Press, 1986, London.