

Heat Conduction in a Thin Rod

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Giacomo Lorenzoni

http://www.giacomo.lorenzoni.name/PEEI_4.0.0.1/heat_conduction_in_a_thin_rod/

http://www.giacomo.lorenzoni.name/PEEI_4.0.0.1/PEEIapplDown.aspx?var=3

Heat conduction in a thin rod

This text is integrating part of the homonymous link in [PEEI: a computer program for the numerical solution of systems of partial differential equations](#).

Coordinate system: Cartesian

System of measurement: International System of Units

Coordinates of Cartesian system: \underline{x}_v of which $\underline{x}_v = \{x, y, z\}$ $[x] = [y] = [z] = [\text{length}]$
 $\mathcal{R}(x) = \mathcal{R}(y) = \mathcal{R}(z) = (-\infty, \infty)$

Unknown functions: $\tau(\underline{x}_v)$ of which $[\tau] = [\text{temperature}]$

Differential analytic model: $\partial^2 \tau(\underline{x}_v) / \partial x^2 + \partial^2 \tau(\underline{x}_v) / \partial y^2 + \partial^2 \tau(\underline{x}_v) / \partial z^2 = 0$

Definition set: $\{\underline{x}_v / x_1 \leq x \leq x_2; y_1 \leq y \leq y_2; z_1 \leq z \leq z_2\}$ $x_1 = y_1 = z_1 = 0$ $x_2 = 100$ $y_2 = z_2 = 1$.

Conditions: $\tau(x_1, y, z) = \tau(x_2, y, z) = 1000$ $K \cdot (\partial \tau(x, y_1, z) / \partial y) - H \cdot (\tau(x, y_1, z) - \tau_\infty) =$
 $K \cdot (\partial \tau(x, y_2, z) / \partial y) + H \cdot (\tau(x, y_2, z) - \tau_\infty) = K \cdot (\partial \tau(x, y, z_1) / \partial z) - H \cdot (\tau(x, y, z_1) - \tau_\infty) = K \cdot (\partial \tau(x, y, z_2) / \partial z) +$
 $H \cdot (\tau(x, y, z_2) - \tau_\infty) = 0$ $K = 20$ $H = 50$ $\tau_\infty = 0$

Solution in [1]: $\tau(\underline{x}_v) = (\tau(x_1, y, z) \cdot \sinh(\mu \cdot (L - x)) + \tau(x_2, y, z) \cdot \sinh(\mu \cdot x)) / \sinh(\mu \cdot L)$ $\mu^2 = 4 \cdot H / K$ $L = x_2 - x_1$

Related files: [mad.txt](#)

Case 1:

Related files: [points_1.txt](#), [PEEI_mem_1.bin](#), [cond_1.txt](#), [PEEI_sol_1.txt](#), [plot_1.jpg](#)

Case 2:

Related files: [points_2.txt](#), [PEEI_mem_2.bin](#), [cond_2.txt](#), [PEEI_sol_2.txt](#), [plot_2.jpg](#)

Case 3:

Related files: [points_3.txt](#), [PEEI_mem_3.bin](#), [cond_3.txt](#), [PEEI_sol_3.txt](#), [plot_3.jpg](#)

Case 4:

Related files: [points_4.txt](#), [PEEI_mem_4.bin](#), [cond_4.txt](#), [PEEI_sol_4.txt](#), [plot_4.jpg](#)

Case 5:

Related files: [points_5.txt](#), [PEEI_mem_5.bin](#), [cond_5.txt](#), [PEEI_sol_5.txt](#), [plot_5.jpg](#)

Case 6:

Related files: [points_6.txt](#), [PEEI_mem_6.bin](#), [cond_6.txt](#), [PEEI_sol_6.txt](#), [plot_6.jpg](#)

Case 7:

Related files: [points_7.txt](#), PEEI_mem_7.bin, [cond_7.txt](#), [PEEI_sol_7.txt](#), [plot_7.jpg](#)

Case 8:

Related files: [points_8.txt](#), PEEI_mem_8.bin, [cond_8.txt](#), [PEEI_sol_8.txt](#), [plot_8.jpg](#)

Case 9:

Related files: [points_9.txt](#), PEEI_mem_9.bin, [cond_9.txt](#), [PEEI_sol_9.txt](#), [plot_9.jpg](#)

Case 10:

Related files: [points_10.txt](#), PEEI_mem_10.bin, [cond_10.txt](#), [PEEI_sol_10.txt](#), [plot_10.jpg](#)

Bibliography:

[1] H. S. CARSLAW, J. C. JAEGER, *Conduction of Heat in Solids*, second edition, Oxford University Press, 1986, London.